and explored several subcategories that are especially important in astronomy: thermal energy, gravitational potential energy, and mass-energy. Now we are ready to return to the question of where objects get their energy. Because energy cannot be created or destroyed, objects always get their energy from other objects. Ultimately, we can always trace an object's energy back to the Big Bang [Section 1.1], the beginning of the universe in which all matter and energy is thought to have come into existence.

For example, imagine that you've thrown a baseball. It is moving, so it has kinetic energy. Where did this kinetic energy come from? The baseball got its kinetic energy from the motion of your arm as you threw it. Your arm, in turn, got its kinetic energy from the release of chemical potential energy stored in your muscle tissues. Your muscles got this energy from the chemical potential energy stored in the foods you ate. The energy stored in the foods came from sunlight, which plants convert into chemical potential energy through photosynthesis. The radiative energy of the Sun was generated through the process of nuclear fusion, which releases some of the mass-energy stored in the Sun's supply of hydrogen. The mass-energy stored in the hydrogen came from the birth of the universe in the Big Bang. After you throw the ball, its kinetic energy will ultimately be transferred to molecules in the air or ground. According to present understanding, the total energy content of the universe was determined in the Big Bang. It remains the same today and will stay the same in the future.

4.4 THE UNIVERSAL LAW OF GRAVITATION

Newton's laws of motion describe how objects in the universe move in response to forces. The laws of conservation of momentum, angular momentum, and energy offer an alternative and often simpler way of thinking about what happens when a force causes some change in the motion of one or more objects. However, we cannot fully understand motion unless we also understand the forces that lead to changes in motion. In astronomy, the most important force is gravity, which governs virtually all large-scale motion in the universe.

MA™ **Motion and Gravity Tutorial, Lesson 2**

What determines the strength of gravity?

Isaac Newton discovered the basic law that describes how gravity works. Newton expressed the force of gravity mathematically with his **universal law of gravitation**. Three simple statements summarize this law:

- Every mass attracts every other mass through the force called *gravity.*
- The strength of the gravitational force attracting any two objects is *directly proportional* to the product of their masses. For example, doubling the mass of *one* object doubles the force of gravity between the two objects.

MATHEMATICAL INSIGHT 4.2

Mass-Energy

It's easy to calculate mass-energies with Einstein's formula $E = mc^2$. Once we calculate an energy, we can compare it to other known energies.

EXAMPLE: Suppose a 1-kilogram rock were completely converted to energy. How much energy would it release? Compare this to the energy released by burning 1 liter of oil.

SOLUTION:

Step 1 Understand: We are asked how much energy would be released by converting all the mass of a 1-kilogram rock to energy. We therefore need to know the total mass-energy of the rock, which we can then compare to the energy released by burning a liter of oil.

Step 2 Solve: The total mass-energy of the rock is given by $E = mc^2$, where m is the 1-kilogram mass and $c = 3 \times 10^8$ m/s:

$$E = mc^2 = 1\text{ kg} \times \left(3 \times 10^8 \frac{\text{m}}{\text{s}}\right)^2$$
$$= 1\text{ kg} \times \left(9 \times 10^{16} \frac{\text{m}^2}{\text{s}^2}\right)$$
$$= 9 \times 10^{16} \frac{\text{kg} \times \text{m}^2}{\text{s}^2}$$
$$= 9 \times 10^{16}\text{ joules}$$

We now compare this energy to the energy released by burning 1 liter of oil, which is 12 million joules (see Table 4.1). Dividing the mass-energy of the rock by the energy released by burning 1 liter of oil, we find

$$\frac{9 \times 10^{16}\text{ joules}}{1.2 \times 10^7\text{ joules}} = 7.5 \times 10^9$$

Step 3 Explain: We have found that converting a 1-kilogram rock completely to energy would release 9×10^{16} joules of energy, which is about 7.5 billion times as much energy as we get from burning 1 liter of oil. Moreover, by looking it up, we can find that 7.5 billion liters of oil is roughly the amount of oil that *all* cars in the United States use in a week. Thus, complete conversion of the mass of a single 1-kilogram rock to energy could yield enough energy to power all the cars in the United States for a week. Unfortunately, no technology available now or in the foreseeable future can release all the mass-energy of a rock.

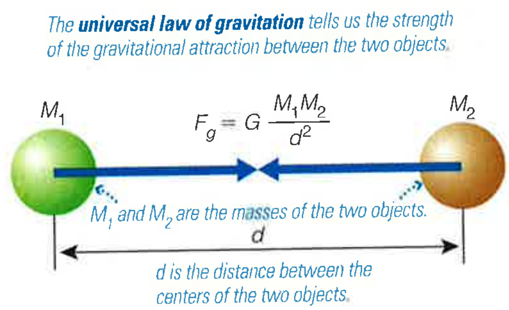

FIGURE 4.16 The universal law of gravitation is an *inverse square law*, which means that the force of gravity declines with the *square* of the distance *d* between two objects.

- The strength of gravity between two objects decreases with the *square* of the distance between their centers. We therefore say that the gravitational force follows an **inverse square law**. For example, doubling the distance between two objects weakens the force of gravity by a factor of 2^2, or 4.

These three statements tell us everything we need to know about Newton's universal law of gravitation. Mathematically, all three statements can be combined into a single equation, usually written like this:

$$F_g = G\frac{M_1M_2}{d^2}$$

where F_g is the force of gravitational attraction, M_1 and M_2 are the masses of the two objects, and d is the distance between their centers (Figure 4.16). The symbol G is a constant called the **gravitational constant**, and its numerical value has been measured to be $G = 6.67 \times 10^{-11}\ \text{m}^3/(\text{kg} \times \text{s}^2)$.

THINK ABOUT IT

How does the gravitational force between two objects change if the distance between them triples? If the distance between them drops by half?

MA **Orbits and Kepler's Law Tutorial, Lessons 1–4**

How does Newton's law of gravity extend Kepler's laws?

By the time Newton published *Principia* in 1687, Kepler's laws of planetary motion [Section 3.3] had already been known and tested for some 70 years. Kepler's laws had proven so successful at predicting planetary positions that there was little doubt about their validity. However, there was great debate among scientists about *why* Kepler's laws hold true.

Newton solved the mystery by showing that Kepler's laws are consequences of the laws of motion and the universal law of gravitation. For example, we've already seen how we can understand Kepler's second law, that a planet moves faster when it is closer to the Sun, by thinking about conservation of angular momentum. Kepler's third law, that average orbital speed is higher for planets with smaller average orbital distance, comes directly from the fact that planets closer to the Sun have a stronger gravitational attraction to the Sun. In essence, Newton used the mathematical expressions of his laws of gravity and motion to show that a planet orbiting the Sun should automatically have an elliptical orbit that obeys Kepler's laws.

Newton's explanation of Kepler's laws sealed the triumph of the Copernican revolution. Prior to Newton, it was still possible to see Kepler's model of planetary motion as "just" another model, though it fit the observational data far better than any previous model. By explaining Kepler's laws in terms of basic laws of physics, Newton removed virtually all remaining doubt about the legitimacy of the Sun-centered solar system.

In addition, Newton found that Kepler's laws were only part of the story of how objects move in response to gravity. Remember that Kepler had discovered his laws by analyzing the orbits of planets around the Sun, and he therefore had no reason to think that his laws would apply to other cases, such as moons orbiting planets or comets orbiting the Sun. However, when Newton analyzed his equations for gravity and motion, he discovered that Kepler's laws are just one special case of a more general set of rules about orbiting objects. These more general rules explain the motion of objects throughout the universe, and they are a crucial part of modern astronomy. Let's explore four ways in which Newton extended Kepler's laws.

Planets Are Not the Only Objects with Elliptical Orbits Kepler wrote his first two laws for planets orbiting the Sun, but Newton showed that any object going around another object will obey these laws. For example, the orbits of a satellite around Earth, of a moon around a planet, and of an asteroid around the Sun are all ellipses in which the orbiting object moves faster at the nearer points in its orbit and slower at the farther points.

Ellipses Are Not the Only Possible Orbital Paths Kepler was right when he found that ellipses (which include circles) are the only possible shapes for **bound orbits**—orbits in which an object goes around another object over and over again. (The term *bound orbit* comes from the idea that gravity creates a *bond* that holds the objects together.) However, Newton discovered that objects can also follow **unbound orbits**—paths that bring an object close to another object just once. For example, some comets that enter the inner solar system follow unbound orbits. They come in from afar just once, loop around the Sun, and never return.

More specifically, Newton showed that the allowed orbital paths are ellipses, parabolas, and hyperbolas (Figure 4.17a). Bound orbits are ellipses, while unbound orbits can be either parabolas or hyperbolas. Together, these shapes are known in mathematics as the *conic sections*, because they can be made by slicing through a cone at different angles (Figure 4.17b). Note that objects on unbound orbits still obey the basic principle of Kepler's second law: They move faster when they are closer to the object they are orbiting, and slower when they are farther away.

Objects Orbit Their Common Center of Mass We usually think of one object orbiting another object, like a planet orbiting the Sun or the Moon orbiting Earth. However,