Solutions to exam review questions.

1. To find the luminosity, we use $L = 4\pi d^2 F$, where d is the distance to the system and F is the flux. First find the distance and its uncertainty:

$$d = \frac{1}{p}$$

where d is in parsecs and p in arcseconds. Thus,

$$d = \frac{1}{p} = \frac{1}{0.02143 \text{ pc}} = 46.7 \text{ pc}.$$

The uncertainty is given by

$$\sigma_d = \sqrt{\left(\frac{\partial d}{\partial p}\right)^2 \sigma_p^2} = \left|\left(\frac{\partial d}{\partial p}\right) \sigma_p\right| = \left|\frac{\sigma_p}{p^2}\right|$$

Thus, we have

$$\sigma_d = \frac{0.00286}{0.02143^2} = 6.2 \text{ pc}.$$

Changing units, $d = 46.7 \pm 6.2$ pc $= (1.44 \pm 0.19) \times 10^{20}$ cm.

Now the uncertainty on the luminosity L is

$$\sigma_L = \sqrt{\left(\frac{\partial L}{\partial d}\right)^2 \sigma_d^2 + \left(\frac{\partial L}{\partial F}\right)^2 \sigma_F^2} = 4\pi\sqrt{(2dF\sigma_d)^2 + (d^2\sigma_F)^2}$$

(Notice that each of the terms in the expression has dimensions of d^2F, *i.e.*, of luminosity—a good sign.)

Now, just plug in the numbers:

$$\begin{aligned}\sigma_L &= 4\pi\sqrt{(2dF\sigma_d)^2 + (d^2\sigma_F)^2} = \\ &4\pi\Big\{\Big[2(1.44 \times 10^{20} \text{ cm})(3.3 \times 10^{-12} \text{ erg cm}^{-2} \text{ s}^{-1})(1.9 \times 10^{19} \text{ cm})\Big]^2 + \\ &\Big[(1.44 \times 10^{20} \text{ cm})^2(5 \times 10^{-13} \text{ erg cm}^{-2} \text{ s}^{-1})\Big]^2\Big\}^{-1/2} = \\ &2.62 \times 10^{29} \text{erg s}^{-1}\end{aligned}$$

So $L = (8.6 \pm 2.6) \times 10^{29}$ erg s^{-1}.

2. The signal-to-noise expression for a CCD image (as shown in class) is

$$\frac{S}{N} = \frac{N_*}{\sqrt{N_* + n_{\rm pix}(N_{\rm sky} + N_{\rm read}^2)}}$$

(This just comes from the same method of partial derivatives as in problem 1, considering three independent noise sources: the star photons, the sky photons in each pixel, and the read noise in each pixel.)

Let's calculate the source counts for each star (which we'll need in part b anyway:

Star	Total ADU	ADU/pixel of sky	# of pixels in aperture	$N_{\rm sky} n_{\rm pix}$ (ADU)	N_* (ADU)	$M_{\rm inst} = -2.5\log(N_*)$
Standard A	50,000	200	30	6,000	44,000	-11.61
Target 1	3,000	200	10	2,000	1,000	-7.50
Target 2	2,500	200	10	2,000	500	-6.75
Standard B	30,000	200	30	6,000	24,000	-10.95

Now convert these numbers from ADU to electrons using the gain of 2 e^-/ADU for the two stars we're concerned with in this part. Now we can easily plug these numbers in the S/N equation, obtaining the numbers in the last column:

Star	$N_{\rm sky} n_{\rm pix}$ (e^-)	N_* (e^-)	$N_{\rm read}^2 n_{\rm pix}$ (e^-)	S/N
Target 1	4,000	2,000	250	25.3
Standard B	12,000	48,000	750	195

b. Instrumental magnitude is just $M_{\rm inst} = -2.5\log(N_*)$, already given in the first table. You could calculate it in terms of electrons rather than ADU as well; it doesn't matter.

c. Note that Target 1 and Standard A were both observed at the same airmass, $x = 1$. That means that we can do this the easy way. For two measurements at the same airmass, the extinction and the instrumental offset both get wrapped up into a single term. The difference between the instrumental magnitudes of Standard A is $true - inst = 17.61$. Therefore, the same is true for Target 1. $y_{\rm Target\ 1} = -7.50 + 17.61 = 10.11$. (As a doublecheck, note that this is a larger number than 6, so this target star is fainter, as it should be based on the number of counts.)

d. For this one, we don't have a standard measurement at the same airmass, so we actually have to figure out the extinction curve. It's pretty easy, though. As noted above, $true \; - inst = 17.61$ for Standard A. We also have $true \; - inst = 6.5 - (-10.95) = 17.45$ for Standard B. Thus, our data points $(x, \Delta M)$ are (1, 17.61) and (2, 17.45). The line through these points has slope 0.16 mag/airmass and intercept 17.77, so these are our extinction coefficient and instrumental offset, respectively. So for Target 2, $y_{\mathrm{Target\ 2}} = M_{\mathrm{inst}}+$ offset $-A_\lambda x = -6.75+17.77-(0.16)(1.5) = 10.78$. Also note that if we reduce Target 1 in the same way, we get $y_{\mathrm{Target\ 1}} = M_{\mathrm{inst}}+$ offset $-A_\lambda x = -7.50 + 17.77 - (0.16)(1) = 10.11$, in agreement with our answer to part c.

e. Color corrections are less of a concern simply because the Stromgren filters are so much narrower in wavelength span than the Johnson filters. Therefore, you have less of a change in extinction across the bandwidth of the filter, and thus the effective "filter profile" (the filter convolved with the atmospheric extinction) mostly just changes in height rather than changing drastically in shape for observations at different airmasses.

f. Target 1 was observed at an airmass of 1. The only place in the sky with an airmass of 1 is the zenith. Therefore, Target 1 crosses the zenith in Tempe. Thus, its declination is 33°.